

LITERATURE CITED

1. B. F. Gordiets, A. I. Osipov, and L. A. Shelepin, Kinetic Processes in Gases and Molecular Lasers [in Russian], Nauka, Moscow (1980).
2. V. D. Rusanov and A. A. Fridman, Physics of Chemically Active Plasma [in Russian], Nauka, Moscow (1984).
3. T. E. Andreeva, S. I. Gritsinin, et al., "Relaxation of vibrationally excited nitrogen with gas-dynamic phenomena," *Kratk. Soobshch. Fiz.*, No. 7 (1983).
4. A. A. Vedenov, S. V. Drobyazko, et al., "Influence of acoustic waves generated in the discharge region on the operation of a pulse CO₂ laser in repetitive mode," *Teplofiz. Vys. Temp.*, 13, No. 2 (1975).
5. B. F. Gordiets and Sh. S. Mamedov, "The distribution function and the velocity of relaxation of the vibrational energy in a system of anharmonic oscillators," *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 3 (1974).
6. K. V. Baiadze, V. M. Vetsko, et al., "Anomalous heating of nitrogen in discharge," *Fiz. Plazmy*, 5, No. 4 (1979).
7. G. M. Batanov, S. I. Gritsinin, et al., "Uhf discharge at high pressure," *Tr. Fiz. Inst. Akad. Nauk SSSR*, 160 (1985).
8. Yu. S. Akishev, A. V. Dem'yanov, et al., "Determination of the constants of vibrational transfer in N₂ by way of heating of the gas," *Teplofiz. Vys. Temp.*, 20, No. 5 (1982).
9. C. E. Treanor, I. W. Rich, and R. G. Rehm, "Vibrational relaxation of anharmonic oscillators," *J. Chem. Phys.*, 48 (1968).
10. A. Yu. Zakharov and V. I. Turchaninov, "Program STIFF for solving stable systems of ordinary differential equations," *Instruktsiya IPM AN SSSR*, Moscow (1977).
11. A. A. Rukhadze, V. P. Silakov, and A. V. Chebotarev, "Propagation of shock waves in vibrationally excited nitrogen," *Kratk. Soobshch. Fiz.*, No. 6 (1983).
12. A. I. Zhamakin and A. A. Fursenko, "On a certain monotonic difference scheme of direct calculation," *Zh. Vychisl. Mat. Mat. Fiz.*, 20, No. 4 (1980).
13. R. W. MacCormack, "The effect of viscosity in hypervelocity impact cratering," *AIAA Pap.*, No. 69-354, New York (1969).

PHENOMENOLOGICAL MODEL OF FIRST-ORDER PHASE TRANSITIONS IN
A DEFORMABLE ELASTIC MEDIUM

I. G. Getts, A. M. Meirmanov,
and N. V. Shemetov

UDC 536.421

We assume that the continuous medium under consideration can be described with the help of averaged quantities and that these quantities satisfy the laws of conservations of mass, momentum, and energy [1]

$$\partial\rho/\partial t + \operatorname{div}(\rho\mathbf{v}) = 0; \quad (1)$$

$$\partial/\partial t(\rho\mathbf{v}) + \operatorname{div}(\rho\mathbf{v}\otimes\mathbf{v} - \mathbf{P}) = \rho\mathbf{f}; \quad (2)$$

$$\partial/\partial t(\rho(U + (1/2)|\mathbf{v}|^2)) + \operatorname{div}(\rho(U + (1/2)|\mathbf{v}|^2)\mathbf{v} - \kappa\Delta\Theta - P(\mathbf{v})) = \rho\mathbf{f}\cdot\mathbf{v} + \rho g \quad (3)$$

everywhere in the region $\Omega \subset \mathbb{R}^3$ occupied by the medium for all values of the time t in the interval $(0, T)$ for any state of the continuous medium. Here ρ is the density; \mathbf{v} the velocity; \mathbf{P} the symmetric stress tensor; U the specific internal energy; Θ the temperature; κ the thermal conductivity; \mathbf{f} the external body force density; g the internal heat source density. Equations (1) through (3) have the form of the abstract conservation law $\partial A/\partial t + \operatorname{div}(A\mathbf{v} - \varphi) = X$, and when the functions A , \mathbf{v} , and φ have first-order discontinuities it is necessary to use the integral identity $\iiint_{\Gamma} (A(\mathbf{l} + \mathbf{v}) - \varphi)\mathbf{v}d\Gamma = \iiint_G XdG$ for an arbitrary volume G of the four-dimensional region $\Omega_T = \Omega \times (0, T)$ enclosed by a smooth surface Γ whose outward normal is \mathbf{v} (\mathbf{l} is a unit vector along the time axis).

Novosibirsk. Translated from *Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki*, No. 6, pp. 43-50, November-December, 1987. Original article submitted January 20, 1987.

It is evident that the system (1) through (3) is not closed. If the medium were in a single state, the system of equations could be closed by the equation of state and the axioms of thermodynamics. For example, an elastically deformed state of a continuous medium is completely described by the Gibbs identity

$$\Theta dS = dU - (1/\rho)P(I - 2E)^{-1} : dE \quad (4)$$

and the equation of state, where the free energy $F = U - \Theta S$ is defined to be a known isotropic function of the thermodynamic parameters E and Θ , where $E = (1/2)(I - T^{-1}T^{*-1})$ is the Euler deformation tensor and T is the distortion tensor [1]. This construction, when applied to the system (1) through (3), determines the dependence $P = P(E, \Theta)$ (for example, the Duhamel-Neuman law) and $U = U(E, \Theta)$.

In order to more clearly present the essence of the reasoning used in the present paper, we limit ourselves for the time being to a single spatial variable. In this case the Gibbs identity can be written either in the form $\Theta dS = dU - p dV$ ($V = 1/\rho$ is the specific volume; p is the stress), or as

$$d\Phi = -S d\Theta - V dp \quad (5)$$

($\Phi = U - \Theta S - pV$ is the thermodynamic Gibbs potential). If Φ is a known function of the independent parameters Θ and p , then $S = -\partial\Phi/\partial\Theta$, $V = -\partial\Phi/\partial p$.

Suppose the system is found in two different states (phases I and II), and the dependence of the potential Φ on Θ and p is known in each phase. The phenomenological description of first-order phase transitions has been discussed in detail in [2] for a two-parameter medium, and following the system of axioms assumed there, one can determine regions corresponding to each phase in the plane of the independent thermodynamic variables Θ and p . According to the first postulate, at the points of contact of the different phases of the continuous medium, the stress and temperature of the medium are continuous. This means that in the (Θ, p) -plane the regions corresponding to the different phases share a common boundary, which is called the phase equilibrium curve. The explicit form of the phase equilibrium curve is determined from the condition that the Gibbs thermodynamic potential be continuous at the points of phase equilibrium: $\Phi^I(\Theta, p) = \Phi^{II}(\Theta, p)$, and this is the content of the second postulate. The typical shape of the phase equilibrium curve is shown in Fig. 1a. Let the equation of this curve be $\Theta = \Theta_*(p)$; then the function $\Theta_*(p)$ is usually called the melting temperature. The term "equilibrium thermodynamics" applied to the case considered here means that values of the parameters Θ and p lying above the phase equilibrium curve always correspond to phase I, and values of these parameters below this curve always correspond to phase II. Knowing the dependence of U and V on Θ and p in each phase (it is possible that these functions are not smooth when passing through the phase equilibrium curve), it is possible to formally close the system of equations (1) through (3).

In actual fact, however, this construction does not completely close the system (1) through (3). We illustrate this fact on the simple example of a one-parameter medium, when motion of the medium is not taken into account. A model describing phase transitions in a medium where the only independent thermodynamic variable is the temperature is known as the Stefan problem. This problem is normally formulated not as a problem of finding the solution of an integral identity corresponding to the law of conservation of energy (where the velocity is set equal to zero and the density is a known constant; such a solution is called a generalized solution), but as a consequence of this identity, when the structure of the solution is assumed to be known a priori. In particular, it is assumed that the solution has a strong discontinuity, where in the space defined by the physical variables (x, t) there exists a sufficiently smooth surface Γ (to be determined as part of the solution) dividing the region Ω_T into two subregions, each of which is occupied by a single phase. If the dependence of U on Θ is linear everywhere except for the melting point $\Theta = \Theta_*$ (at this point the function U is not defined and has a first-order discontinuity) then it follows from the integral identity that the temperature satisfies the (possibly inhomogeneous) heat equation in each subregion of Ω_T . The so-called Stefan condition on the surface Γ [1] also follows from this integral identity. The equations in each subregion, the Stefan condition, the condition of thermodynamic equilibrium $\Theta = \Theta_*$ on the boundary Γ , the initial conditions, and the boundary conditions on the boundary of the region Ω , together form the classical Stefan problem, and the corresponding solution is usually called classical, as well. It is obvious that any classical solution will also be a generalized solution, but the converse is not true in general.

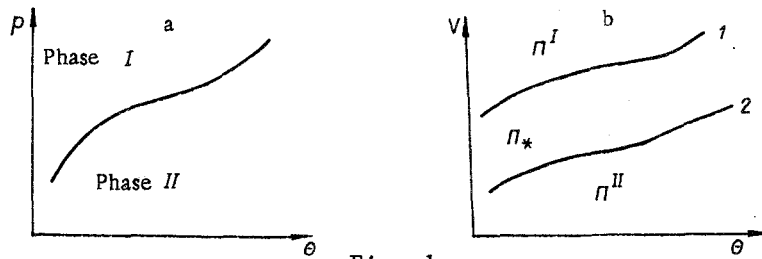


Fig. 1

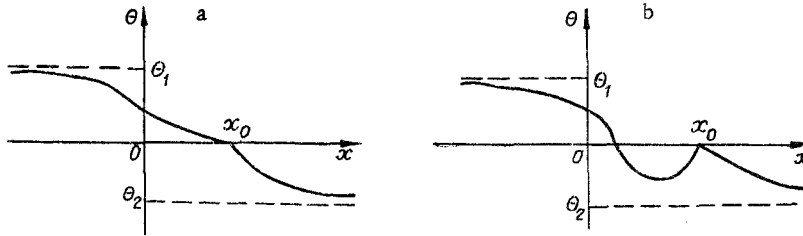


Fig. 2

As an example, we consider the cooling of an infinite cylindrical bar whose melting temperature is $\Theta_* = 0$. The bar is moved with a constant velocity v_0 from an insulated furnace occupying the half-space $x < 0$ and having the constant temperature $\Theta_1 > 0$, to a refrigerator occupying the half-space $x > 0$ and having the temperature $\Theta_2 = \text{const} < 0$ [3, p. 193]. We assume that the law of cooling $\partial\Theta/\partial n = \gamma(\varphi - \Theta)$ ($\varphi = \Theta_1$ for $x < 0$ and $\varphi = \Theta_2$ for $x > 0$) is obeyed at the surface of the bar, that the temperature of the bar is constant in any section perpendicular to its axis, and that the cooling process is stationary in a coordinate system fixed to the nonmoving furnace. Then the classical Stefan problem has the form (for notational simplicity all appropriate constants are set equal to unity)

$$v_0 d\Theta/dx - d^2\Theta/dx^2 + \Theta = \varphi, \quad x \neq x_0, \quad (6)$$

$$\Theta(x_0) = 0, \quad d\Theta/dx(x_0 - 0) - d\Theta/dx(x_0 + 0) = v_0, \quad \lim_{|x| \rightarrow \infty} |\Theta - \varphi| = 0.$$

When $\Theta_1 + \Theta_2 > 0$ the solution $\Theta(x)$ and x_0 are written out explicitly. When $v_0 \leq v_* = |\Theta_2| \cdot (1 + |\Theta_2|)^{-1/2}$ the solution has the structure assumed initially (Fig. 2a): $\Theta(x) > 0$ for $x < x_0$, $\Theta(x) < 0$ for $x > x_0$. If, however, $v_0 > v_*$, then the solution is singular (Fig. 2b): there exists a point x_1 such that the function Θ is strictly negative in the interval (x_1, x_0) . Since the point x_0 is the phase boundary and the liquid phase is always to the left of this point, it would appear at first glance that the problem defined in (6) describes the complicated phenomenon of supercooling. But the mathematical model only describes that which is built into it by the physical axioms. And the Stefan problem is formulated from the axioms of equilibrium thermodynamics, in which the liquid phase always corresponds only to points of the continuous medium with temperatures higher than or equal to the melting temperature. Where then is the error? The error was introduced in the mathematical hypothesis about the structure of the solution in passing from the generalized formulation in terms of an integral identity to the classical formulation (6).

Before one can analyze the original integral identity, it is necessary to correctly determine its solution. For the cylindrical bar problem the integral identity has the form

$$(v_0 U - d\Theta/dx) \Big|_a^b + \int_a^b (\Theta - \varphi) dx = 0, \quad (7)$$

where (a, b) is an arbitrary interval $-\infty < a < b < \infty$. The identity (7) contains the required functions U and Θ and is closed by the equation of state, in which U is a known smooth function of the temperature Θ except at the point $\Theta = \Theta_*$, where it is not defined and has a first-order discontinuity. But how can U be found if the region occupied by the continuous medium has zero temperature? Recall that we assumed that any state of the continuous medium could be described in terms of averaged quantities. Two methods can be used. The first is purely mathematical [4] and assumes that in this state the energy U can have any value in the interval $[U(\Theta_* - 0), U(\Theta_* + 0)]$ ($U(\Theta_* \pm 0)$ are the limiting values of the internal energy to the left and right of the point of discontinuity $\Theta = \Theta_*$); the second method is purely physical

[5] and assumes that both phases coexist at each point in this state of the continuous medium and that all of the thermodynamic quantities depend continuously on a new parameter – the fraction of the liquid phase. In the framework of equilibrium thermodynamics, it turns out that these two methods are equivalent and form the basis of the following physical axiom: there exist states of the continuous medium for which Θ is identically equal to Θ_* , while U takes all values in the interval $[U(\Theta_* - 0), U(\Theta_* + 0)]$.

It is natural to call such a state of the medium the transitional phase, differing from the liquid and solid phases. The inverse dependence of the temperature on internal energy can be found following the arguments discussed above: Θ depends smoothly on U everywhere outside the interval $[U(\Theta_* - 0), U(\Theta_* + 0)]$ (for example, linearly), is identically equal to Θ_* inside this interval, and is continuous for all values of U . It is easy to distinguish the phases in terms of the function U , even when $\Theta = \Theta_*$: the liquid phase corresponds to $U \geq U(\Theta_* + 0)$, the solid phase to $U \leq U(\Theta_* - 0)$, and the transitional phase to $U(\Theta_* - 0) < U < U(\Theta_* + 0)$. If now U is regarded as the solution of (7), then all quantities forming this identity will be determined correctly. A detailed analysis of the identity (7) shows that when $v_0 > v_*$, its unique solution describes the liquid phase for $x \leq x_1$, the solid phase for $x \geq x_2$, and the transitional phase for $x_1 < x < x_2$ [3, p. 218].

Hence in order to solve the problem correctly it was necessary to expand the point $\Theta = \Theta_*$ (in the space of the independent variable Θ) into a segment defining the transitional state by introducing a new independent thermodynamic variable, the specific internal energy U .

Returning to the original system of equations of motion of the continuous medium, we see in the case of a single spatial variable that the phase equilibrium curve in the plane of the thermodynamic variables Θ and p can be thought of as having a fold, which hides the transitional state. This fold is smoothed out if we transform to new independent variables Θ and V , or U and p . We assume that the phase equilibrium curve (see Fig. 1a) can be specified in the form $p = p_*(\Theta)$, and we consider the new independent variables Θ and V . In terms of these variables, phase I corresponds to the points of region Π^I , lying above curve 1 ($V = V_*^I(\Theta)$), phase II corresponds to the points of region Π^{II} , lying below curve 2 [$V = V_*^{II}(\Theta)$], and the transitional phase corresponds to the points of region Π_* lying between these curves (Fig. 1b). Explicit forms of the curves $V_*^I(\Theta)$ and $V_*^{II}(\Theta)$ can be obtained by substituting the value $p = p_*(\Theta)$ in the equations of state $V = V^I(p, \Theta)$ and $V = V^{II}(p, \Theta)$ in each of the pure phases. The possibility of such a description of the metastable transitional state was indicated in [2, p. 310], assuming the independent variables V and Θ . Finally, the transitional state is actually unstable, if it occurs at all, but since one of the fundamental principles of the phenomenological theory of continuum mechanics is the assumption that any state of the continuous medium can be described in terms of averaged quantities, the complete description of the transitional phase is necessary in order to close the model of phase transitions considered here.

Recall that the assumption of the existence of a transitional phase, allowing the fold running along the phase equilibrium curve in the (Θ, p) -plane to be straightened, was the first of the newly added postulates of the model. The second postulate, which should not cause any objection since it is a consequence of the law of conservation of energy, is the assumption that the Gibbs identity (5) is also valid in the transitional phase. The third postulate assumes that the thermodynamic potential Φ is continuous everywhere in the region Π of the variables Θ and V .

It turns out that the second and third postulates completely determine Φ , U , S , p , and F as functions of the independent parameters Θ and V everywhere in the region Π_* . A given point M on the phase equilibrium curve $p = p_*(\Theta)$ corresponds to the segment $I(\Theta) = \{(\Theta, V) \in \Pi_* | \Theta = \text{const}\}$ in the region Π_* of the variables Θ and V , and $p = p_*(\Theta) = \text{const}$ and $\Phi = \text{const}$ along this segment. The latter assertion follows from the Gibbs identity (5) in Π_* . The condition that the function Φ is continuous everywhere in Π uniquely determines its constant value on the segments $I(\Theta)$. The free energy density $F = \Phi + pV$ is continuous everywhere in Π since it is a combination of continuous functions. Let $F^I(V_*^I(\Theta), \Theta)$ and $F_*(V_*^I(\Theta), \Theta)$ be the limiting values of the function $F(V, \Theta)$ on the curve $V = V_*^I(\Theta)$ from the regions Π^I and Π_* , respectively. Then differentiating the equation $F^I(V_*^I(\Theta), \Theta) = F_*(V_*^I(\Theta), \Theta)$ and using the relations $S = -\partial F / \partial \Theta$, $p = \partial F / \partial V$, we obtain the result that the entropy S is also continuous through the line $V = V_*^I(\Theta)$. The line $V = V_*^{II}(\Theta)$ is considered in the same way. The continuity of U everywhere follows from the relation $U = \Phi + \Theta S + pV$.

Therefore if the free energy density F is known in regions Π^I and Π^{II} , then it is known everywhere in region Π and it is continuous there along with its derivatives $\partial F/\partial\theta = -S$ and $\partial F/\partial V = p$.

The thermal conductivity κ appears in the law of conservation of energy (3) and can be found in each phase (I or II). That is, it can be considered a given function of the variables θ and p , possibly having a first-order discontinuity through the phase equilibrium curve $p = p_*(\theta)$. It would be difficult to produce an argument which would determine the thermal conductivity in region Π_* of the new variables θ and V , therefore we assume that κ is a continuous function of the parameters θ and V everywhere in region Π , and assume that it is a linear function of V on the segments $I(\theta)$ in Π_* . This is the postulate used in [5] in determining the characteristics of the two-phase region of Borisov. This last assumption is the fourth of the newly introduced postulates and completely closes the model of phase transitions in a deformable elastic medium for the case of a single spatial variable. The initial postulates of the model are consistent with the experimental relation between pressure and deformation for all deformations for first-order phase transitions in metals [6, p. 540].

The principles used in constructing a mathematical model of first-order phase transitions with a single spatial variable carry over completely to problems involving a large number of dimensions. It is true that in general the number of independent thermodynamic parameters of a deformable elastic medium is greater than two, and additional axioms (besides those already introduced) are therefore necessary to close the model. We consider only small deformations of the medium, a linear dependence of the stress tensor on the deformation tensor, and a linearized form of the Gibbs identity:

$$\theta dS = dU - P : dE. \quad (8)$$

In terms of the free density $F = U - \theta S$ the identity (8) becomes

$$dF = -Sd\theta + P : dE, \quad (9)$$

This identity, plus the assumption that F is an isotropic function of the deformation tensor E , and the assumption of a linear relation between the stress tensor and the deformation tensor implies that in each phase the equation of state $F = F(\theta, e_1, e_2)$ ($3e_1 = \text{tr } E$, $(9/2)e_2^2 = 3E : E - (\text{tr } E)^2$) and the identity (9) completely determine the thermodynamic state of the medium as a function of the three independent parameters θ, e_1, e_2 . Therefore each phase is a three-parameter medium, and hence the transitional phase between the two observable phases can also be considered as a three-parameter medium.

We choose the temperature θ and the invariants $\sigma_1 = (1/3) \text{tr } P$, $\sigma_2 = (1/\sqrt{2}) \sqrt{3P : P - (\text{tr } P)^2}$ of the stress tensor as the independent parameters of the thermodynamic state of the medium. If we assume that θ and σ_1, σ_2 are continuous at points of the continuous medium in which the different phases are in contact, then in the space of the independent thermodynamic parameters $(\theta, \sigma_1, \sigma_2)$ the phase equilibrium surface dividing regions corresponding to the different phases is an equilibrium state the explicit form of the phase equilibrium surface can be obtained if we require that the function $\Phi = U - \theta S - P : E$ be continuous at the points of the contact of the phases:

$$\Phi^I(\theta, \sigma_1, \sigma_2) = \Phi^{II}(\theta, \sigma_1, \sigma_2).$$

We assume that the phase equilibrium surface Γ is specified by the equation (Fig. 3a)

$$\sigma_2 = \varphi(\sigma_1, \theta). \quad (10)$$

According to the principles discussed earlier, the introduction of the new independent thermodynamic parameters $\theta, e_1,$ and e_2 allows one to unfold the phase equilibrium surface Γ into a transitional state region Π_* separating the regions Π^I and Π^{II} corresponding to phases I and II (Fig. 3b). The postulate on the validity of the Gibbs identity

$$d\Phi = -Sd\theta - 3e_1 d\sigma_1 - e_2 d\sigma_2 \quad (11)$$

in terms of the thermodynamic potential ϕ and the invariants of the stress and deformation tensors, or the same identity, but written for the free energy density

$$dF = -Sd\theta + 3\sigma_1 de_1 + \sigma_2 de_2,$$

along with the postulates that the potential ϕ and the pressure σ_1 are continuous everywhere in region Π of the thermodynamic parameters $\theta, e_1,$ and e_2 , completely determines the dependent thermodynamic variables $\phi, F, U, S, \sigma_1, \sigma_2$ in the transitional phase.

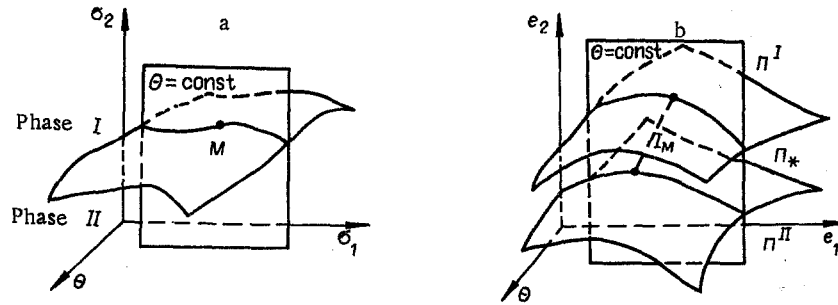


Fig. 3

We consider sections of the set Π_* formed by the planes $\{\Theta = \text{const}\}$ in the form

$$Q_\Theta = \{(e_1, e_2, \Theta) \in \Pi_* \mid \Theta = \text{const}\}.$$

It is known [7] that for nearly all values of a , the set of surfaces $\{\Phi_*(e_1, e_2, \Theta) = a\}$ constitutes a finite number of smooth straightened curves which are either closed, or whose end-points pass out on the boundary of the set I_Θ dividing the sets

$$\{\Phi(e_1, e_2, \Theta) > a\}, \{\Phi(e_1, e_2, \Theta) < a\}$$

and are such that

$$(\partial\Phi_*/\partial e_1)^2 + (\partial\Phi_*/\partial e_2)^2 > 0$$

$[\Phi_*(e_1, e_2, \Theta)]$ is the value of Φ in region Π_* and is assumed to be the minimum of a twice-differentiable function]. We show that all connected elements of the set of surfaces are in fact line segments joining the rim of the section Q_Θ . These properties of the surface lines mean that we can introduce the curvilinear coordinates (ξ, η) in a small neighborhood of each connection of the elements of the set of surfaces, where $\xi = \Phi_*(e_1, e_2, \Theta)$. Returning to the identity (11) and Eq. (10), we see that $\sigma_1 = \sigma_1(\xi)$ in each of these neighborhoods. It then follows that we have the relation $1 = d\Phi_*/d\xi = -[3e_1 + e_2 \partial\varphi(\sigma_1(a), \Theta)/\partial\sigma_1] (d\sigma_1(a)/d\xi)$ on the lines of the surfaces $\{\Phi_* = a\}$. Hence the linear combination of the variables e_1 and e_2 is constant along each connection of the lines.

Let M be a point on the phase equilibrium surface Γ in the space of the variables $(\sigma_1, \sigma_2, \Theta)$, and let I_M be a line segment in Π_* , where the mapping $(\sigma_1, \sigma_2, \Theta) \rightarrow (e_1, e_2, \Theta)$ maps M into I_M . Then $\Phi_*(e_1, e_2, \Theta) = \Phi^I(M) = \Phi^{II}(M)$, $(e_1, e_2, \Theta) \in I_M$. The free energy density F is uniquely determined in terms of the potential Φ , and F is also a continuous function of the independent thermodynamic variables (e_1, e_2, Θ) everywhere in region Π . In addition, the derivatives of F with respect to e_1, e_2 , and Θ are also continuous in Π since the derivatives $\partial F/\partial e_1 = 3\sigma_1$ and $\partial F/\partial e_2 = \sigma_2$ are continuous by construction, and the continuity of the derivative $\partial F/\partial \Theta = -S$ can be proven by analogy with the one-dimensional case. The postulates that the thermal conductivity κ is continuous everywhere in region Π and that κ depends linearly on e_1 in region Π_* along the line segments for which $\Phi_* = \text{const}$ close the model.

Returning to the model as a whole, we note that the continuity of Θ, σ_1 , and Φ at the points of contact of the phase cannot cause any objection, but the postulate that σ_2 is continuous is not supported by any additional theoretical reasoning or experimental facts. The authors are unaware of a theoretical or experimental paper where equilibrium phase transitions in a three-parameter deformable elastic medium have been analyzed, especially by the construction of a phase equilibrium surface in the space of the independent thermodynamic variables. However, starting from the Gibbs identity (11), and carrying through the analogy with a multicomponent medium [2], where the chemical potentials of the components of the mixture and the temperature appear as differentials in the Gibbs identity and are continuous upon changes in the aggregate state of the medium, it is natural to require that all thermodynamic variables appearing as differentials in (11) be continuous at phase equilibrium points, including the invariant σ_2 . Since the variation of the deformation is plastic in the transitional region on the lines of the surfaces of the potential Φ , an indirect support of the correctness of the model is the existence of a plastic region between the two pure phases in solid-phase transformations [8].

Remark 1. A model of phase transitions in an ideal two-parameter medium with a spherical stress tensor can be constructed in analogy with the case of a single spatial variable.

Remark 2. The initial and boundary conditions are dictated by the structure of the differential equations, and the formulation of initial and boundary-value problems is the subject of a separate publication.

LITERATURE CITED

1. L. V. Ovsyannikov, Introduction to Continuous Mechanics [in Russian], Novosibirsk State University, Novosibirsk (1977), Chaps. I, II.
2. L. D. Landau and E. M. Lifshits, Statistical Physics [in Russian], Nauka, Moscow (1964).
3. A. M. Meirmanov, The Stefan Problem [in Russian], Nauka, Novosibirsk (1986).
4. O. A. Oleinik, "On a method of solving the general Stefan problem," Dokl. Akad. Nauk SSSR, 135, No. 5 (1960).
5. V. T. Borisov, V. V. Vinogradov, and I. L. Tyazhel'nikova, "Quasiequilibrium theory of the two-phase region and its application to hardening of alloys," Izv. Vyssh. Uchebn. Zaved., Chern. Met., No. 5 (1977).
6. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian], Fizmatgiz, Moscow (1963).
7. A. S. Kronrod, "On functions of two variables," Usp. Mat. Nauk, 5, No. 1 (1950).
8. A. P. Chupakhin, A. A. Sidel'nikov, and V. V. Boldyrev, "Effect of mechanical stress induced by solid-phase transformations on kinetics," Izv. Akad. Nauk SSSR (Siberian Branch), Ser. Khim. Nauk, No. 6 (1985).

UNSTEADY THREE-DIMENSIONAL LAMINAR BOUNDARY LAYER ON BLUNT BODIES WITH STRONG BLOWING

S. V. Peigin and B. F. Filonenko

UDC 532.526

One must investigate unsteady heat and mass transfer in flow of a compressible gas over blunt bodies with a permeable surface in order to solve many applied problems. In particular, these problems arise unavoidably and in general are time-dependent when gas is blown through a porous or perforated surface in order to form a gas curtain. Similar questions arise also in examining a number of chemical technology facilities in various regimes of operation.

For these reasons the literature has a number of papers in which both approximate analytical methods [1, 2] and numerical methods [3-5] have been used to study unsteady processes occurring in laminar planar or axisymmetric boundary layers in a compressible gas on a permeable surface. The influence of blowing (or suction) on the characteristics of the unsteady two-dimensional boundary layer was examined in [6, 7]. Unsteady heat transfer in the vicinity of a stagnation point with two radii of curvature was the subject of [8, 9], and the influence of strong blowing on the basic characteristics of steady flow in a three-dimensional laminar boundary layer was examined in [10-13].

This paper has obtained numerical and asymptotic solutions, over a wide range of variation of the governing parameters, of the equations of the unsteady three-dimensional laminar boundary layer on a permeable surface, including the case of strong blowing.

1. Statement of the Problem. We consider three-dimensional unsteady flow of a supersonic gas over blunt bodies with a permeable surface at large incident stream Reynolds number Re . We choose a nondegenerate curvilinear coordinate system (x^1, x^2, x^3) with origin at the stagnation point, and normally related to the wetted surface: $x^3 = \text{const}$ is a family of surfaces parallel to the body surface ($x^3 = 0$), and x^1 and x^2 are curvilinear coordinates on the surface.

Later we shall also investigate bodies for which the longitudinal pressure gradient ∇p^* obtained by solving the equations describing inviscid flow over a given body is a quantity of order $O(\rho_\infty V_\infty^2/L)$. As is shown by asymptotic analysis of the unsteady three-dimensional Navier-Stokes equations for the case of hypersonic flow over bodies with blowing present and under the conditions

Tomsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 50-56, November-December, 1987. Original article submitted September 9, 1986.